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ABSTRACT

This study, concerning the conceptual and econometric problems involved in estimating educational production functions, focuses on the following topics: 1) the meaning of an educational production function estimated from cross-section data; 2) the measurement of the output of schools; 3) the problem of measuring the initial endowment of students upon entering school; 4) the measurable dimensions of learning environment, both school and home; 5) the shortcomings of the Project Talent five-year follow-up data; and 6) estimated educational production functions, using Project Talent data as well as data from the Equal Educational Opportunity Survey of the Office of Education. Some major findings are: a) the estimated relationships are consistent with the conceptual model developed in this report; b) teacher quality appears to be an important determinant of scholastic success; and c) the production functions explain a very small percentage of the variance of scholastic achievement, even using the full range of social class and school input variables. (Author/LS)

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Educational Production Function

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Final Report
on
Research Supported by the U.S.
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LIST OF TABLES

Summary

This study concerns the conceptual and econometric problems involved in estimating educational production functions. Attention is given to the following topics. First, the meaning of an educational production function estimated from cross-section data is discussed. Attention is given to the problem of simultaneity, and to the difficulties arising from the absence of the usual maximizing behavioral assumptions which ordinarily underlie production function estimates. Second, I deal with the measurement of the output of schools. In this study, I concentrate on achievement scores as a measure of output, although there is some attention given to economic measures, such as post-school earnings. Third, I discuss the problem of measuring the initial endowment of students upon entering school. A method of dealing with the normal mis-specification of educational production functions arising from this source is developed and implemented. Fourth, the measurable dimensions of the learning environment, both school and home, are discussed. A model of the learning environment is developed, based on the findings of sociological and psychological research. Fifth, the shortcomings of the Project Talent five-year follow-up data are discussed. Particularly important is the magnitude of non-response to the follow-up and non-response on particular items by those included in the follow-up. Sixth, educational production functions are estimated using Project Talent data, as well as

data from the Equal Educational Opportunity Survey of the Office of Education. The following findings are particularly important: a) the estimated relationships are consistent with the conceptual model developed in this report; b) teacher quality appears to be an important determinant of scholastic success; c) some other dimensions of the school environment appear to be important, although the relationships are somewhat inconsistent; and d) the production functions explain a very small percentage of the variance of scholastic achievement, even using the full range of social class and school input variables.

I. PREFACE

The following is the final report on the first stage of a research project on educational production functions supported by the U.S. Office of Education under grant number OEC 1-7-000451-2651.

The first stage has been devoted to an exploration of the conceptual and econometric problems involved in the construction of educational production functions, and the estimation of some preliminary functions in which the school output is measured by scholastic achievement. It was originally planned that the data used in the first stage would be drawn entirely from Project Talent. However, unforeseen delays in acquiring the necessary tapes have resulted in a somewhat restricted use of Talent data supplemented in part by data from the Office of Education's Equality of Educational Opportunity Survey.

The second stage is devoted to the economics of educational production functions. Here the emphasis will be on the relationship between school inputs, social class, and post school earnings, occupational attainment, and employment status. Some exploratory analysis in this area was included in stage one. The preliminary results are outlined in the postscript to this report. The data used in this economic analysis are from Project Talent.

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[1]During the period of this contract (11/1/66 through 12/31/68), I have received support from other sources for study in these problems. This report constitutes a summary of my progress. To identify particular findings or concepts with specific sources of funding is, of course, pointless and impossible.

I would like to thank a number of people for assistance. Pamela King, Alice Crampin, Mathew Lambrinides, and Keith Clemens served capably as research assistants. I received useful advice from Zvi Griliches, Susan Contratto, Christopher Jencks, Lester Thurow, Stephan Michelson , Henry Levin, Arthur MacEwan, Thomas Weiskopf, and Christopher Sims. Terry Rothra and Deanna Lee typed the manuscript

Researchers using Project Talent data are asked to include the following statement in their report: "This investigation utilized the Project Talent Data Bank, a cooperative effort of the U.S. Office of Education, the University of Pittsburgh, and the American Institutes for Research. The design and interpretation of the research reported herein, however, are solely the responsibility of the author."

II. INTRODUCTION

An educational production function relates school and student inputs to a measure of school output. Representation of the educational production process in this form is of particular interest in the descriptive study of human capital formation as well as in normative investigations of the optimal allocation of resources in the educational sector.

If schooling has any unique effect on labor productivity or earnings, this effect should be traceable to the development of cognitive skills and attitudes as a consequence of school attendance. Further, we may be able to relate the development of productive personal attributes to school policies concerning the allocation of scarce educational resources. A production function relating school inputs to the development of an individual's productive capacity would give us a much better idea why the better-educated earn more. Moreover, by investigating differences in production functions for different racial and social class groups, as well as differences in educational inputs among these groups, we may better understand one important aspect of the determination of the distribution of personal earnings.

In the determination of school policy, and in long-run educational planning, knowledge of the educational production function is essential to the achievement of efficient resource allocation. This is true, of course, regardless of whether the decision unit is pursuing the objective of growth, equality, or any combination of these and other goals. Without an estimate of the technology of education, the relation between the opportunity costs of particular policies and their expected benefits must be little more than guesses.

An educational production function is defined as follows:

(1) $$A = f(X_1, \ldots, X_m, X_n, \ldots, X_v, X_w, \ldots, X_z)$$

where

A = some measure of school output -- for example, a score on a scholastic achievement battery;

$X_1, \ldots, X_m$ = variables measuring the school environment. The variables here would typically include the amount and quality of teaching services, the physical facilities of the school, the length of time that the student is exposed to these inputs;

$X_n, \ldots, X_v$ = variables representing environmental influences on learning outside the school -- e.g., the parents' educational attainment;

$X_w, \ldots, X_z$ = variables representing the initial level of learning attained by the student prior to entry into the type of schooling in question.

We are interested in gaining estimates of the structural parameters of the function, f. It will be seen below that we cannot estimate the above equation in the form presented, although some progress can be made with a slightly modified version.

The data at our disposal are ordinarily based on a cross-section of students. Although I will dwell at some length on the deficiencies of our data, the information available for the estimation of educational production functions is, in many respects, superior to that underlying production function estimates in the economy. The crucial deficiency, it will be seen, is not so much in the absence of data as in the absence of a theory of the learning process which will guide us in the process of estimation. The engineering processes used in the production of physical commodities are reasonably well understood. They suggest appropriate specifications of the production function, as well as some a priori limits on what are regarded as plausible estimates. In the estimation of educational production functions, the psychologist replaces the engineer or agronomist as the source of technical information on the production process. Despite some fruitful developments in learning theory, we are left without much guidance for the underlying technical processes involved.

Nonetheless, to preview some of our results, it will be seen that a reasonable a priori model of the production of scholastic achievement can be specified on the basis of existing theory. Moreover, preliminary estimates of this function are encouraging.

Attempts to measure the relationship between school inputs and outputs have occupied the attention of a number of educational researchers over the last half-century. Yet the estimation of the structural parameters of a production function similar to (1) is relatively new.[2] The results of the studies completed to date are difficult to summarize, in part because of the large variety of measurements used, and in part because of the diversity of findings. In any case, the purpose of this paper is not primarily to present empirical estimates of production functions, but

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[2]As I will refer to the methods and results of studies in the course of the paper, I will briefly review them now. (All works referred to in this report appear in the bibliography.)
Herbert Kiesling used data generated by the Quality Measurement Project of New York State to estimate school production functions for various communities in New York. Martin Katzman estimated production functions for a variety of school outputs of elementary schools in Boston. As a part of the study which gave rise to the report of the Central Advisory Council for Education (the Plowden Report) in England, G. F. Peaker estimated a series of production functions for British elementary education. Thomas Fox and John Holland and Jesse Burkhead have estimated production functions for a wide range of school outputs for Atlanta and Chicago, as reported in Burkhead. I have not included in this list the study of Finis Welch, as he relies on highly aggregated inputs and his estimates can only be identified as educational production functions by some stretch of the imagination. Eric Hanushek and David Armor have used U.S. data on the sixth grade to estimate production functions for elementary education. The International Project for the Evaluation of Educational Achievement, under the direction of Torsten Husén, has estimated similar functions for the determination of mathematics achievement in a sample of 12 countries. A considerable amount of additional work is now in progress.

rather to explore some of the conceptual and econometric problems involved in this type of estimation. Nonetheless, the results of some of these studies, as well as my own results, will be introduced as illustrations.

Part III will include a discussion of the behavioral assumptions underlying the usual production function estimation, and the particular difficulties encountered when the concepts are applied to schools. Parts IV and V are devoted to the measurement of school outputs and student inputs of the production process. The measurement and interpretation of school inputs is discussed in Part VI; and in Part VII the statistical properties of the Project Talent data are surveyed. In Part VIII some results based on Project Talent data are presented. These results are compared with estimates based on EEOS data in Part IX. Problems of specification bias are discussed in Part X. Part XI is a brief conclusion.

III. ESTIMATING A PRODUCTION MODEL FOR SCHOOLS

The striking characteristic about the production process in schools is the degree to which it appears to be complex, unsystematic, or just plain not understandable. In a statistical investigation using non-experimental data, the most we can expect is discovery of some of the relationships among measured dimensions of the process based on the particular configuration of data in our sample. We are thus limited both by the preconceptions of the researchers responsible for the selection of the sample and the available data as well as the patterns of variation which school decision-making processes have brought about in the sample of schools chosen. To use the apt analogy of Marshak and Andrews, we are not in the position of the agronomist who seeks to understand production relations in agriculture with a mind to making agriculture more productive. He can experiment, varying his factor inputs systematically and in any desired combination, and thus, under ideal conditions, predict the likely consequences of changes in factor inputs on productivity. Nor are we in the position of the meterologist who relies on non-experimental data, but seeks only to predict normal behavior rather than to effect events. We have the worst of these worlds, for we seek to affect the pattern of educational output by altering school inputs, and yet our data are generated entirely by systems of decision-making and student responses entirely beyond our control. Thus

Thus we are faced with the usual problem of simultaneous equation bias which has plagued the estimation of production functions at the firm level:[4] any single equation approach to the estimation of (2) will yield inconsistent estimates of the structural parameters f_j.

One possible way out of this difficulty arises from the basic implausibility of the above behavioral model. It may be that school administrators do not select school inputs as if they were maximizing any well defined function of school outputs. This seems a reasonable assumption, given that school administrations know very little about the underlying technology and are subject to a wide variety of political and legal constraints. In this case we can take the X_{ji} as exogenous for the purposes of estimation.

Rejection of an optimizing decision model for school administrators relieves us of at least one simultaneity problem (there will be others), but it deprives us of the usual interpretation of the estimated parameters of (2) as a production function. We ordinarily reserve for this concept a relation which indicates the maximum output consistent with a given set of inputs. Yet if school administrators conform to no systematic optimizing behavioral model, then the observations on which our estimates are based are not generally <u>technically</u> efficient. Thus we arrive

[4] See Marshak and Andrews, and Nerlove for a discussion.

our ability to calculate the consequences of departures from existing ways of producing education is very limited indeed.[3]

But the limited variation in the configuration of inputs of our sample of schools is just the beginning of the difficulty. Postponing the discussion of the precise functional form to be used, assume for the moment that we seek to estimate the production function (1) in the form

$$A_i = f_0 + f_1X_{1i} + f_2X_{2i} + \ldots + f_zX_{zi} + u_i \tag{2}$$

where

A_i = the achievement score (or other output measure) for the i^{th} student

$f_0,\ldots,f_z$ = the parameters of the production function to be estimated

X_{ji} = the amount of input j (including measures of home environment) devoted to observation of i's education, $j = 1\ldots z$

u_i = the disturbance term

Yet we may expect that the school inputs are endogenous to some system, for example, a system of equations based on the school administrators social welfare function, the educational production function(s), and an educational budget constraint.

[3] A considerable amount of educational research has used experimental techniques. See, for example, Gray and Klaus, and Kirk. These methods hold out some promise for empirical determination of the educational production function.

at some sort of average production function. Only if the absolute degree of inefficiency is uncorrelated with the level of factor inputs (which seems unlikely) will the estimates f_j from (2) represent unbiased estimates of the true underlying production relation.[5]

While the determination of school inputs can perhaps plausibly be regarded as exogenous to our system, one set of inputs most certainly must be taken as endogenous -- student attitudes toward themselves and toward learning. These are both important determinants of achievement and a consequence of the students' past and present achievement levels, as well as other influences. In this case simultaneity seems unavoidable. Estimates based on (1), including student attitudes as explanatory variables, will in general be correlated with the disturbance term. Our solution is to estimate (2), an equation in which attitudes are excluded, the explanatory variables being confined to those which are exogenous. This reduced form equation incorporates the effects of attitudes indirectly as they are related to the set of exogenous variables. Unless we are interested in increasing scholastic achievement by directly affecting student

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[5] Of course, the constant term will be biased downward. If we had a number of different observations on inputs for the same school, we could use school dummy variables to eliminate this "management bias." See Massell and Hoch.

attitudes, little is lost by excluding the attitude variables from the equation.[6]

The dirth of knowledge concerning the underlying learning relationships makes a priori specification of a functional form for the estimation of educational production relations particularly difficult. The notion of diminishing marginal product is an appealing one, although certainly not well established in the field of education. From this standpoint a function linear in the logarithms of the variables would seem somewhat superior. The possibility of positive interactions between inputs also recommends this form. Nonetheless, the restrictions of the Cobb-Douglas function are severe -- particularly important to my mind is the fact that the cross derivatives among any pair of inputs, each of which is positively related to output, must also be positive. This would require, for example, that increases in the quality of teachers are more effective among the children of well-educated parents. For reasons of simplicity, in the work below I will use the linear additive form presented in (2) above.[7]

Not all children learn the same way or the same things. Lesser and Stodolsky, for example, found dramatic differences in the patterns of scholastic proficiency on four different learning dimensions among Chinese, Jews, Negroes, and Puerto

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[6]Nonetheless, in my results I present both the reduced form and the (biased) estimates of the structural equation itself. See Part X.

[7]Hanushek found that the logarithmic form gave slightly better significance for the estimates of the parameters of his produc-

Ricans. When we find consistent differences in patterns of response to school inputs, we have good grounds for grouping our students according to these systematic patterns and estimating a number of different technologies. Although I know of no work presenting systematic statistical tests of the hypothesis that educational production functions estimated from sub-populations were drawn from the same underlying population, casual inspection of the results of Hanushek, Kiesling, and my own work strongly suggest that it is useful to think in terms of distinct educational production technologies, at least for black and white and rich and poor students, separately.[8]

If we may take a lesson from the study of economic growth, we should anticipate that the major changes in productivity of school resources will come from changes in production functions, including changes in relations between home background and achievement, as well as the more conventional input-output relations. If this is our goal, we should seek to identify 'best practice' schools and develop a quantitative explanation of their superior technique.

7cont. tion functions.

8 In my results below I have estimated functions for black 12th grade students separately, sometimes with a regional stratification.

IV. SCHOOL OUTPUTS

We are interested in the economic consequences of schooling. Thus our output measures ideally should concern economic and social behavior following the termination of schooling. Characteristically, we are forced to use indices of student 'achievement' based on tests administered while the youth is still enrolled. These achievement scores must be considered either proxies for, or perhaps influences on, post-school economic behavior. Scholastic achievement is presumably not valued per se, but only as an intermediate input into other valued measures of performance. Thus although we will here use achievement, A, as the output measure, our rationale for doing this is a social welfare function, many of whose arguments are themselves functions of scholastic achievement.

Although the evidence of a relationship between scholastic achievement and earnings is not well established, we proceed on the assumption that scholastic achievement has economic consequences, at least for some major groups of workers.[9]

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[9] See Hansen, Scanlon and Weisbrod; Duncan, and Part XII of this report for some evidence on this question.

Scholastic achievement, of course, is not a single dimension of school output. Literally hundreds of instruments have been devised to measure achievement in school. And achievement as ordinarily defined on these tests is but one aspect of the consequences of schooling on the growth of cognitive skills and personality. In addition to the effect of achievement on economic performance in the post-school years, we may be interested in the effects of schooling directly on an individual's self confidence, self-concept, or his sense of control over his environment. Evidence of zero order correlations among individual test scores, some of which are presented in Table 1, suggests that the relations among at least some of these measures are rather weak.

Thus the output of schools is multidimensional with a vengance, and to complicate matters, there are no convenient sets of 'prices' with which to aggregate the output. Of course, few problems would arise if we found that the technologies for the production of each dimension of the output were roughly similar. This, however, does not seem to be the case. Estimates of the reduced form equation (2) in which the dependent variable is a measure of scholastic achievement, differ considerably from estimates in which

Table 1

Zero-Order Correlations Among Measures of School Outputs.

Twelfth Grade Boys,

U.S.

	2	3	4	5	6
1. Information Total	.23	.65	.76	.54	.19
2. Self-Confidence		.17	.19	.09	.11
3. English Total			.67	.46	.26
4. Mathematics Total				.57	.20
5. Abstract Reasoning					.19
6. Clerical Checking					

The test scores are described in Project Talent, Flanagan (1964).

an index of the student's sense of control over his environment is the dependent variable. Production function estimates for different types of scholastic achievement differ also (see Part VIII).

Apparently we require not just one production function, but many, which, along with given resource endowments and budget constraints, could determine a production possibility set for the school. The production possibility set, along with a social welfare function indicating the relative importance of the various dimensions of school output, would then form the analytical basis for resource allocation in the school.[10]

For the purposes of policy making, we are particularly interested in the structural parameters of the production function (2), for under ideal conditions they may be interpreted as the marginal products of the inputs in question, that is, $MP_j = \partial A / \partial x_j = f_j$. We may use this information to move in the direction of optimal input proportions as defined by the conditions[11]

[10] The social welfare function would presumably reflect a combination of societal, parental and child interests.

[11] Of course, we are here accounting for only one output.

$$(3) \qquad \frac{\partial A/\partial X_j}{\partial A/\partial X_k} = \frac{\hat{f}_j}{\hat{f}_k} = \frac{p_j}{p_k} \qquad \text{for all pairs, } j, k$$

However, difficulties arise when we seek to compare the marginal products of the same input for two different groups of students. We find, for example, that the estimate of the structural parameter relating to the verbal ability of teachers as an input into an achievement production function is considerably greater for black twelfth graders in the U.S. than for whites. Can we infer from this that verbally adroit teachers ought to be shifted from white to black districts?

The output measure is ordinal; there is no zero point and no well defined unit of measurement for achievement.[12] Thus, while the marginal rate of substitution in production -- represented in the additive linear form by the ratio of regression coefficients of any two input factors -- is still a valid analytical concept; the absolute magnitude of the marginal product is not. Among students scoring at very different parts of the scale of measurement, equal units of increase in scores are not comparable; for example, it may be "easier" to make gains at the lower end of the scale than at the upper end due to a so-called 'ceiling effect.' We really need to know the relationship

[12] At least one writer has constructed a cardinal index of achievement based on the size of vocabulary (Bloom, pages 103-104) Whether words known is linearly related to anything important is not known. For the concept of a zero point, see Thurstone.

between our output measure, A, and measures of directly desired performance, such as earnings.

Although there is some evidence of a linear relation between achievement and earnings, it is certainly not sufficient to justify much confidence in a cardinal interpretation of measures of school learning.

A further problem remains. Our output indices are subject to some error -- that is, test score = "true measure" + error, and, consequently, var(test score) = var(true measure) + var (error). We have no idea of the validity of the test -- that is, its correlation with a hypothetical true measure. But some idea of the magnitude of the error may be gained from estimates of the reliability of the tests. The reliability of our tests is in the neighborhood of .9.[13] Taking this as an upper estimate of the validity, at least 19 per cent of the variance of the test scores is due to test errors. Assuming that the errors in test measurement are uncorrelated with our explanatory variables, even if our explanatory variables predict the true measure with perfect accuracy, a validity of .9 imposes an absolute maximum proportion of variance explained by our equations of .81. It will be seen below that the actual R^2's are considerably lower.

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[13] Although there are various ways of measuring test reliability, we may convey the essential meaning as the zero order correlation between scores on the odd and even number questions of the same test or the zero order correlation between two versions of the test given to the same individual at roughly the same time.

V. MEASURING THE STUDENT INPUT

An achievement score must be considered a measure of gross output. Our goal is to estimate the relationship between school inputs and net output, or value added. For this we need a measure of the raw material inputs, i.e., student ability, or, alternatively, the level of learning upon entry to the school in question.

The problem is that all measures of relevant student 'ability' depend heavily on previous learning, and are hardly distinguishable from measures designed explicitly to test scholastic achievement. Intelligence, as measured by the standard I.Q. instruments, is a developmental concept for measuring general learning.[14] Moreover, most I.Q. tests depend heavily on verbal facility, which is probably a good reflection of general school learning, and which apparently develops similarly in all children.[15] Evidence that 'abilities' measured in IQ tests are in large measure a product of the educational environment is suggested by Table 2, based on a study of identical twins who were separated prior to age three. Over 60 percent of the variance of differences in IQ can be explained by differences in the educational environment. The physical and social environment together explain less than a third of the variance of the IQ differences. There is substantial further evidence on the lack

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[14] See Hunt.

[15] Bloom, pp. 71 and 104.

Table 2

The Effect of Environmental Differences on I.Q. Differences of Identical Twins Reared Apart[a]

Environmental Difference	Effect[b]	t statistic
Educational	.66	4.2
Social	.25	1.6
Physical	.19	1.3
R^2	.70	
d.f.	16	

a) Data from Newman, Freeman, and Holzinger (1937).

b) Normalized Regression Coefficient of the Environmental Difference in an equation predicting I.Q. differences.

of independence between measures of ability and school learning.[16]

If 'ability' is not an operational concept in this context, how do we intend to interpret the raw material input of the schooling production process? As we are interested in measuring school learning, it would seem reasonable to use tests of learning administered at grade one as a measure of raw input. Beacuse these first grade tests clearly measure the combined effects of genetic ability and environmental influences prior to age six, they are exactly what we need. Thus, our basic equation is:

(4) $A_{12} = f(X_1, \ldots, X_v, A_1)$.

where subscripts on the achievement variable refer to the grade at which the test is taken. In order to estimate a function of this type, we need individual test scores for students at two different levels of schooling. While some data of this type is currently available, and more is on the way,[17] we are generally forced to rely on cross-sections.

If (4) is the correctly specified relation, and we are forced to work with data which do not include the first grade scores (A_1), we may be able to estimate the unbiased regression coefficients of (4) if we have independent evidence on $b_{1,12}$, the regression coefficient of A_1 in equation (4), as well as the estimated equations:

(5) $A_{12} = f^{12}(X_1, \ldots, X_v)$

(6) $A_1 = f^{1}(X_1, \ldots, X_v)$

The unbiased estimates of the regression coefficients of (4)

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[17] from Project Talent, for example

are then

(7) $b^*_i = \hat{b}_i^{12} - b_{1,12}\hat{b}_i^1$

where

$\hat{b}_i^{12}$, $\hat{b}_i^1$ are the estimated regression coefficients of X_i in equations (5) and (6), respectively.

This approach is equivalent to Theil's method of estimating the bias due to specification error.[18]

I have assumed that the relationship between first grade and twelfth grade scores is such that a student scoring one standard deviation above the mean at grade one will, ceteris paribus, score .5 standard deviations above the mean at grade 12. Thus,

(8) $b_{1,12} = .5(\frac{\sigma_{12}}{\sigma_1})$

where

σ_1, σ_{12} = the standard deviation of achievement scores at grades 1 and 12, respectively.

This figure is somewhat arbitrary. It is based on two sets of data. First, longitudinal studies of scholastic achievement scores suggest a simple correlation between early and late scores in the neighborhood of .6 to .9. Most of the studies cover substantially less than twelve years, so we may suspect that the simple correlation of scores at grade one and twelve

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[18] Theil (1957) Our method is based on the assumption that the function, f^1, accurately represents the relationship between each X_i and first grade scores which prevailed at the time of their school entry, and that the vector $X_1, \ldots, X_v$ is the same for a given student at grades one and twelve.

would be somewhat lower.[19] Moreover, the simple correlation is not the appropriate evidence, as we seek an estimate of the partial effects of differences in A_1 on A_{12}. To the extent that students who initially score high on tests are exposed to a better learning environment, the size of the above reported correlations exaggerate the normalized partial relationship between initial endowments and later scholastic achievement.

The second set of data shows that group scores of students classed by socio-economic categories show roughly constant patterns over the years of school. Groups who begin school a standard deviation below the mean end up twelve years later in roughly the same relative position.[20] Given the observed differences in the qualtity of the learning environments of these various groups, we may infer that the partial (normalized) relationship between initial scores and 12th grade scores is less than unity. The choice of .5 is maybe too low, and reflects a desire not to overcorrect for specification bias and thus underestimate the importance of social class and home environment in the learning process.[21]

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[19] Based on 41 longitudinal achievement score correlations reported in Bloom, pages 106-109. The correlations for more widely separated years occupy the lower end of this range.

[20] Coleman, et al., Ch. 3.

[21] All of the achievement measures are subject to error. At grade one the reliability of the achievement score used (verbal ability) is .78. If the validity of this score is only slightly below its reliability, the portion of variance in A_1 due to random error is .5. Thus our method is equivalent to assuming that the normalized partial relationship between the true measure of initial endowments

We assume that the function, f^1 (equation (6)), will consist entirely of arguments relating to the social class and home background of the student, since school inputs could hardly effect scores on tests taken at the beginning of grade one.[22]

[22] There is ample evidence that grade one achievement scores are associated with measures of student social class. See Bereiter, Gray and Klaus, Pasamanick and Knoblock.

VI. MEASURING THE LEARNING ENVIRONMENT OF THE SCHOOL AND THE HOME

We aim to estimate the effect of school inputs on the value added of schools. In order to isolate the impact of schools, however, we must specify as fully as possible all of the environmental influences on learning, that is, ideally, the home and the students' peer groups, as well as the school. A complete specification of the model is particularly important in view of the specification bias likely to arise because of the close association found in most samples between school and home environments which are conducive to learning, and vice versa.

We may derive some suggestion of the relative effects on learning of various dimensions of the individual's environment from another study of identical twins reared apart. In this case we use differences in achievement scores (Stanford Achievement Tests) for paired identical twins as the measure of differential learning. The relationship between environment and learning is suggested by Table 3. Even more than the analogous table for IQ differences, the educational environment is of paramount importance. It alone explains more than 80 per cent of the variation in scholastic achievement. While this is hardly surprising, the insignificance of the social and physical environment among genetically equivalent

Table 3

The Effect of Environmental Differences on Scholastic Achievement Differences Among Paired Identical Twins Reared Apart[a]

Environmental Difference	Effect[b]	t statistic
Educational	.899	7.69
Social	.024	0.21
Physical	.001	0.01
R^2	.82	
d.f.	15	

a) Based on data of Freeman, Newman, and Holzinger (1937).

b) Normalized regression coefficient of the environmental difference measure in an equation predicting achievement differences among paired identical twins.

individuals is striking.[23] Of course, the environments in question may have been very poorly measured. Nonetheless, the finding alerts us once again to the dangers of specification bias in equations with no measure of initial endowment, and suggests that much of the importance of social class in school learning apparent from cross-section studies may reflect genetic differences associated with the educational and social characteristics of the student's family.

Let us begin by asking what aspects of the student's environment could have some effect on learning. A brief survey of the literature on learning suggests that the major characteristics of an environment which will effect the development of school achievement (as well as general intelligence) include:

a. the quantity of verbal interaction and communication with adults;

b. the quality of verbal interaction and communication with adults;

c. the motivation for achievement and understanding in the environment;

d. the richness of and degree of opportunity to explore the physical environment.

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[23] Alone they explain only .13 of the variance of achievement differences.

Our measures of these dimensions of the environment are far from adequate, although data do exist which allow us to attempt an empirical implementation based on the above *a priori* specifications. Moreover a number of reasonably well established relations in sociological and psychological research will assist us in implementing the model.

Beginning with the non-school environment, we may represent the quality of the verbal interaction between child and adult by a measure of the educational level of parents or guardians.[24] Family size, as well as the number of adults living at home, provides a measure of the quantity of interaction and communication.[25] If we restrict ourselves to variables which can be regarded largely as exogenous, the motivation for achievement may be indicated by parental attitudes concerning the importance of schooling,[26] as well

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[24] On the importance of language models, see Olim, Hess, and Shipman, and Jackson, Hess, and Shipman.

[25] Anastasi.

[26] Although we are not able to include this variable in our analysis below as we have no adequate measures in our sample, at least one study, which sampled the parents as well as the children, has confirmed the importance of parental attitudes toward schooling--See Peaker. Of course, parental attitudes must depend in some degree on the particular school in which the child is enrolled. Thus parental attitudes are not unambiguously exogenous.

as measures of the potential objective importance of education in the life of the student. The race of the student may, among other things, constitute a measure of these expected returns, for we have compelling evidence that the economic returns to schooling at the elementary and secondary levels are significantly less for black than for white children.[27] The nature of the physical environment of the home may be measured by the quantity of reading material in the home, the parents' occupation or income, or proxies for these variables, such as measures of the quantity of consumer durables in the home. Evidence of a relation between malnutrition (primarily protein deficiency) and learning difficulties suggests that measures of the physical environment may serve as proxy for aspects of the physical development of the child related to learning, particularly for very poor children.

A number of authors have attempted to take account of the home and social environment of the child by stratifying their analysis according to social class.[28] Available evidence

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[27] Weiss, Hanoch. Differences in family interest in schooling and its associated impact on children's motivation is in part a cultural phenomenon, likely to vary among ethnic groups. For convincing evidence in one case, see M. Gross.

[28] For example, see Kiesling and the U.S. Commission on Civil Rights in their study of the effect of racial integration on scholastic achievement.

suggests that although this technique is certainly useful in reducing the multicollinearity among the explanatory variables, it is a thoroughly inadequate representation of the non-school effects on learning. Peterson and DeBord, for example, found that <u>within</u> two refined sub-strata (white and black lower class urban children in the southern region) variables measuring home environment and parent-child interaction explained .56 (white) and .66 (black) percent of the variance in achievement scores.[29] The predictive power of dimensions of home environment <u>within</u> narrowly defined social strata suggests that an analysis using no other control for social environment will be subject to serious specification bias.[30]

We may proceed in roughly the same manner (although with less confidence) with the empirical implementation of the model of the school environment. The quality of the interaction between adults and child may be represented by measures of the educational level or verbal proficiency of the

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[29] Of course, the Peterson and DeBord findings could result from collinearity between the home environment and school inputs to which the children were exposed. This is not likely to explain the entire result, however. <u>Within</u> a group of black sixth grade students in the third socioecnomic quartile in a large Northeastern metropolitan area, Levin found that, in addition to various school input measures, a number of home measures were significantly related to scholastic achievement. His findings are as yet unpublished.

[30] The strength of the measured relationship between school inputs and achievement observed by Kiesling is probably due in part to this bias.

teachers. The quality of the interaction may depend in some degree on school policies, which may be represented by a host of imperfect measures of such aspects of school environment as the breadth of curriculum, and the amount of extra-curricular activities. The physical environment of the school may be represented by measures of special facilities (labs, libraries, etc.).

Table 4 summarizes our model of environmental influences on learning, and our proposed empirical implementation of the model.

Notice that even this partial specification of the learning environment includes 14 measures, many of which are highly correlated. Thus serious multicollinearity problems arise in the estimation of a full model of the type specified. In order to estimate the above model, we need to reduce the number of variables so as to simplify the presentation and bring the multicollinearity problem within tolerable limits. That is, we would like to replace the equation

(9) $$A = f(x_1, \ldots, x_v)$$

by

(10) $$A = F\left[g_1(x_1, \ldots, x_v),\ g_2(x_1, \ldots, x_v),\ \ldots,\ g_h(x_1, \ldots, x_v)\right]$$

where $h < v$.

Table 4

A Model for the Estimation of the Environmental Influences on Learning

Underlying Influence on Learning	Empirical Representation in the Model: Home	Empirical Representation in the Model: School
1. Quality of Verbal Interaction with Adults	a. education of parents	a. educational level of teachers b. other measures of teacher 'quality', such as verbal ability c. school policies d. teacher attitudes
2. Quantity of Verbal Interaction with Adults	a. family size	a. class size
3. Motivation for Achievement in School	a. parental attitudes toward education b. race, ethnic group c. objective returns to schooling	a. community support of education
4. Richness of the physical environment	a. family income or occupation, consumer durables in the home b. reading material in the home	a. school facilities, labs, libraries, texts, etc.

Thus we may wish to define a new variable, say "teacher quality" as an aggregate of individual variables measuring the teacher's verbal ability, years of schooling experience, certification, and so on. If a significant degree of multicollinearity arose from intercorrelations within the set of variables which form the aggregate variables, the problem will be reduced and the new synthetic variables, represented by $g_1 \ldots g_h$ may be sufficiently orthogonal to allow successful estimation of the relationship. The precise grouping of factors is, of course, determined by more than the desire to reduce multicollinearity, although the usual aggregation rules do not seem particularly helpful here, as we have absolutely no knowledge of the matrix of second derivatives and cross-derivatives which would allow us to make use of them.

We have no previous results or compelling theory which provide guidance in how to aggregate. In situations in which all inputs are priced in the market, and the assumption of maximizing behavior is somewhat more plausible, we ordinarily use factor or commodity prices as the basis of aggregation, as in the measurement of "capital" or intermediate inputs. Failure to appreciate the importance of these assumptions in the validity of any monetary aggregate in production theory has lead to the frequent use of what might be called spurious factors in the analysis of school inputs, such as expenditure per pupil and teachers' salaries. In my own estimates, (for black twelfth graders) instructional expenditure per pupil is in virtually no case significantly related to achievement in a properly specified mode. Yet most of the factors which are purchased with the expenditure, and which account for its variation, such as teacher quality and school facilities, show a strong relationship with achievement. Similarly, whereas teachers' salaries alone explain only .0085 of the variance of achievement, the two factors most closely related to varaiations in teachers' salaries -- teachers' verbal abilities and years of schooling -- explain over four times as much.[31] All of this simply suggests that school administrators are using their resources efficiently as far as the production of scholastic achievement is concerned.[32] Thus the use of monetary aggregates is unfounded in theory.

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[31] In each case I am referring to the increase in the coefficient of determination in an equation already including measures of social background and non-teacher school inputs, as in equation (5) on page 22. See Levin for an analysis of the relation between teacher quality and teacher salary. These two teacher attributes (verbal ability and years of schooling) explain 60 percent of the variance in teachers' salaries.

[32] This inference is supported by a comparison of the estimated marginal products (f_j) and the supply prices for various teacher attributes. (See Levin). Calculations of the cost of unit increase in achievement through increases in each factor based on these estimates show that for the sample under consideration, increases in teacher's verbal ability are more efficient than any other dimension of teacher quality, by a wide margin.

In our situation the best available method seems to be to attempt to identify the underlying dimensions of the input structure, both by a priori and empirical methods. Having done this, we would like to select a variable, or an index based on a number of variables, to represent each dimension. Our a priori specification suggests that we have roughly four important dimensions: teacher quality, teacher quantity, school policy, and physical facilities. One procedure would be to assume that these represent the dimensions of the input structure, and to select from each set a variable to represent the underlying input. Thus it would be plausible to represent teacher quality by the teacher's score on a verbal ability test, at least when we are predicting verbal achievement, and so on.

However, we may combine our preconceptions based on previous research and learning theory with an empirical analysis of the structure of our data, using principal components analysis.[33] Although our results below are generated without the aid of principal components analysis, I am currently experimenting with this approach.

Leaving the problem of aggregation in this unsatisfactory state, let me ask how well we have measured the environmental influences on learning, particularly as they relate

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[33]This is the method used by Kiesling.

to the school. The answer "not very well" stems primarily from three problems: a) our home and school variables fail to capture the complexity and richness of the interactions processes which are relevant to learning; b) we have ignored significant differences in the education offered within the same school; and c) we have measured inputs at only one point of time, while the learning process must certainly be cumulative and therefore depend in some degree on past as well as contemporary inputs.

Turning to the first objection, our measures of social class, family size, class size, teacher quality and school facilities do not measure the quantity and quality of interaction as relevant to learning, but provide only crude measures of a few of the opportunities for it. Two recent studies suggest that the crude measures are a poor substitute for measures of actual observed patterns of interaction. On the basis of highly detailed interviews with 60 parents, both Dave and Wolf found that their measures of home environmental effects on intelligence and achievement explained .57 and .64 of the variance in the attribute measured.[34] The

[34] Recall also the Peterson and DeBord study, op. cit.

crude home environment measures used in our study explain about 10 percent of the variance in individual achievement scores. Presumably analogous studies of actual classroom interaction would reveal that our school measures are a poor representation of our basic learning model.

The second set of problems arises particularly where tracking is widespread and the differences in the education received within the same institution are so great that we really have two or three schools within the same building.[35] Moreover, differences in teacher and administrator attitudes and expectations toward children differ considerably within a school and even within a given classroom.[36] One recent study (Rosenthal and Jacobson) suggests that teacher expectations have a significant effect on learning, at least in the early years of school. The specification error introduced by the failure to measure these within-school and within-classroom differences in inputs is particularly serious because of the correlation of these differences with other of our explanatory variables. Because low social class and minority racial or ethnic status are closely associated with

[35] Differences in the quality and quantity of school inputs received within the same school are documented in Hollingshead.

[36] See Davis and Dollard, pp. 284-285, and Warner, Havinghurst and Loeb, as well as more recent studies by Deutsch and Wilson.

a within-school deprevation of school inputs,[37] the estimates of the parameters reflecting the impact of social class and race are biased upward. Further, because of the serious errors introduced by the school-wide aggregation of the variables measuring school inputs, the estimated effect of the school environment is biased downward.[38]

Our third objection, against the sole use of contemporary inputs measures, would not be serious if children did not move from school to school, and inputs were roughly uniform throughout all of the grades up to the one for which the production function is being estimated. Of course, the world is simply not like that, and I think we sometimes underestimate the seriousness of this problem. In a sample of black sixth grade students in a Northeastern metropolis, 57 per cent had attended more than one school since grade one, and 29 per cent had attended more than two.[39] Evidence from a number of studies of the phasing of learning development over the school years suggests that this problem is particularly serious, as patterns of achievement are apparently established with a high degree of stability in the early grades. Scannell, for example, found that

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[37]See the evidence in Hollingshead and the more recent studies cited in Rosenthal and Jacobson.

[38]In a study in which within-school variations were measured, Peaker found that school inputs were considerably more important in the determination of school achievement (relative to other influences, such as home background) when within-school variations in these inputs were taken into account.

[39]Work in progress by Henry Levin and Stephan Michaelson.

scores on fourth grade tests (Iowa Tests of Basic Skills) explained half the variance in test scores (Iowa Tests of Educational Development) in the twelfth grade.[40] Cardinal measures of scholastic achievement based on vocabulary tests suggest that about two-thirds of what is known in grade twelve was already known in grade six. On the presumption (which seems to have currency among educational psychologists) that the effects of environment on learning are potentially greater during periods in which the most learning takes place, it would seem that measurement of the inputs in the early grades would be essential to the prediction of achievement at the higher levels.[41]

The relative importance of the early years in the learning process suggests one last question: how much impact on measured learning can we expect schools to have? During the elementary and high school years, children ordinarily spend considerably under a quarter of their wakeful hours in school. Moreover, Bloom (1964) suggests that about a third of adult learning is achieved before age six. His survey of the impact of extreme environments on learning suggests that we might expect changes of 1.25 standard deviations on the usual tests due to environment from ages 0 to 18 which is consistently

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[40] Scannell; Bloom summarizes the evidence on the stability of achievement.

[41] In the absence of a time series of school inputs, it might be advisable to concentrate on the estimation of production relations in the early grades.

very conducive or prohibitive to learning. And if the school environment is applicable to an age span in which only two-thrids of the learning takes place, and at that for only part of the time, we might regard an impact of less than a standard deviation as an expected effect of a very good or a very bad school as opposed to an average one.

VII. THE PROBLEM OF NON-RESPONSE

To complete the survey of problems in the estimation of educational production functions, I turn finally to the statistical shortcomings of the available bodies of data. Although I will later make use of data from the Office of Education's Equality of Educational Opportunity Survey, I will not undertake specific analysis of the statistical properties of these data.[42] Rather, I will concentrate on the data from the main sample used here -- the males who were high school seniors in 1960 and who responded to both the initial 1960 Project Talent survey and the five-year follow-up survey. I will consider three distinct sets of problems: non-response of schools in the initial survey; non-response of individual students in the five-year follow-up; and non-response on particular survey items by individuals returning the follow-up questionnaire.

a. Non-response by schools.

The sample is based on a list of public senior high schools (i.e., all schools including a 12th grade) compiled by the Office of Education. These schools were sorted by states and arranged by the nine U.S. Office of Education Regions. The five largest cities -- New York, Chicago, Philadelphia, Detroit, and Los Angeles -- were treated as a separate Region. Within each state,

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[42] The interested reader is referred to Bowles and Levin and Mayeske.

schools were sorted into four size categories:

Category	Number of Students in Grade 12
1	0 - 24
2	25 - 99
3	100 - 399
4	400 and over

Differential sampling ratios were employed in order to get a sufficiently large sample of large schools. A random sample of 1 in 50, 1 in 20, 1 in 20, and 1 in 13 was drawn from each of the four size categories, respectively.[43] Of the 1,063 senior high schools selected in the sample, 987 eventually returned usable data. This 93 percent response rate suggests that school non-response is a relatively minor problem in the Project Talent data.

b. Non-response by individuals in the five-year follow-up.

There were 30,165 male seniors sampled in 1960. Only 15,975 responded to the five-year follow-up survey. The 47 percent non-response rate alone is enough to cast serious doubt about the usefulness of the data. Moreover, we have reasonably good evidence that the pattern of non-response is not random. Table 5 presents data on the distribution of the talent five-year follow-up respondence by race, region, occupation of parents, and urban/rural

[43]The above description is based on Flanagan (1962).

residence. These data are contrasted with the distribution of enrolled 16- and 17-year-olds in 1960, according to the U.S. Census. The correspondence between the Talent definitions and the Census definitions is not exact. Moreover, we are unable to determine the extent to which the discrepancies between the Talent distribution and the Census distributions are the result of biases in the initial Talent sample, as opposed to non-random non-response on the five-year follow-up. Nonetheless, the data in Table 5 do indicate that the males on the Talent five-year follow-up sample are not representative of the total population. The discrepancy for race is particularly serious, there being only half as many blacks in the sample as would have been expected on the basis of a random sample. Of course, the biases indicated in Table 5 may be surmountable by the careful use of a weighting scheme, or preferably by stratification of the sample.

But it would be fortuitous if the non-randomness of response were limited to these variables for which weighting is possible and stratification a plausible procedure. There i strong evidence, for example, that the black respondents to the five-year follow-up are characterized by higher levels of scholastic achievement than would be expected on the basis of a random sample. Data from the Equality of Educational Opportunity Survey (EEOS) allows us to calculate the gap between blacks'

Table 5

Comparison of the Unweighted Project Talent Sample (5-Year Follow-Up of Male Seniors in 1960) with National Population

Characteristic	% in Sample	% in Population[a]
White	.968	.921
North (USOE Regions)[c]	.760	.730
Rich[b] (classified by parents' occupation)	.603	.581

[a]Based on school enrollment of 16- and 17-year-olds. U. S. Census, 1960. PC(2)5A

[b]Occupations in Census:

Rich: skilled worker, foreman; clerical worker; salesman; manager, official; proprietor or owner; professional, technical

Poor: farm owner and/or manager; farm foreman; farm worker; workman, laborer; service worker, including household; protective worker; semi-skilled worker

Occupations in Talent

Rich: skilled worker, foreman; clerical worker; salesman; official; manager; proprietor or owner; technical

Poor: workman, laborer; farm, ranch foreman; farm, ranch worker; private household worker; protective worker; service worker; semi-skilled; don't know

[c]Regions classified as "North": Census: Northeast, North Central, and West

Talent: USOE regions 1, 2, 4, 7, 8 & 9 - New England, Mid-east, Great Lakes, Plains, Rocky Mountains, West, and non-continuous states test scores and the average scores. Similar comparisons may be made with the Talent data, using the reported scores from the 1960 survey and the scores recorded for blacks on the five-year follow-up. These comparisons (using Northern Blacks) are presented in Table 6. The unmistakable inference is that the Talent follow-up sample blacks are achieving much closer to the national mean than are blacks generally.

Table 6
Relative Black and Total Scores,
Equality of Educational Opportunity Survey and Project Talent Five-Year Follow-Up of 1960 Male Twelfth Grade Students

Tests	Difference Between Average Score and Northern Black Score[b] (1)	Standard Deviation of Northern Black Score (2)	Difference, in Black Northern Standard Deviation Units (3) (1)/(2)
Equality of Educational Opportunity Survey[a]			
Verbal Scale Score	16.3	14.4	1.12
Non-Verbal Scale Score	9.1	8.2	1.10
Project Talent			
Reading Comprehension (R250)	4.8	11.5	.41
English Total (R230)	5.4	13.9	.38
Abstract Reasoning (R290)	1.2	3.05	.39
Math I (R311)	2.3	3.4	.67
General Academic Ability (C002)	66.1	120.1	.55

Notes to Table 6

[a]Equality of Educational Opportunity Survey test scores are from the Appendix of the Report.

[b]The average score on the EEOS tests is a weighted average of the total white score and the total black score, using as weights the fraction of white and non-white 16- to 17-year-olds enrolled in school in 1960. The omission of non-black non-whites results in a very slight underestimate of the average score. The average score for the Talent tests is from Flanagan (1964) table 13-2, and refers to all students taking the Talent test battery in 1960. The "Northern" Talent scores refer only to USOE regions 1, 2, and 3. To some small extent, the discrepancy in scores is due to our representation of "Northern" by regions which are roughly "Northeastern." Evidence from the EEOS suggests that black students in the midwest and west in metropolitan areas score about .1 of a standard deviation below blacks in the Northeastern metropolitan areas.

c. Non-response on particular survey items.

Not all of the students returning the five-year follow-up questionnaire furnished all of the requested information. Moreover, there are a substantial number of missing responses on the student information questionnaire administered in 1960. Table 7, which summarizes the extent of the problem, contains information on the number of respondents with no missing data, and those with various numbers of items unanswered. The amount of non-response by questionnaire items is also recorded.

The degree of non-response is substantial, and again, we have compelling evidence that the pattern is not random. Unfortunately, there is no follow-up of the non-respondents from the five-year follow-up. However, for this particular group we can infer a non-random pattern of non-response. A comparison of achievement test scores indicates that those not responding to questions concerning their parents' occupation, education and other dimensions of their social class scored on the average lower than those who did respond.

Number of Respondents by the Number of Missing Observations

1960 Male Seniors in Project Talent Five-Year Follow-Up

Number of Variables with Data Missing (Total Number of Variables = 124)	Cumulative Number of Respondents (Total Number = 15975)
(1)	(2)
1 or less	345
5 or less	2049
10 or less	3373
15 or less	6849
20 or less	10485
25 or less	13571
30 or less	14780
35 or less	15124

Table 8

Number of Respondents with Missing Data on Selected Variables
1960 Male Seniors in Project Talent Five-Year Follow-Up

Total Respondents = 15,975

Variable Number		Number of Cases with Missing Data
7	Class size, science and math	128
10	Senior class size	162
17	Educational innovation	319
21	Starting salary, male BA with no experience	302
25	Percentage of teachers fully certified	198
27	Percentage in college preparatory	841
45	Regular part-time teachers	325
52	Tracking	421
54	Percent of Blacks	1201
67A	Father's occupation	1291
67B	Mother's occupation	1159
68	Father's education	1274
69	Mother's education	1182
71	Own room, desk, typewriter	997
72	Appliances	965
73	TV, telephone, radio, phonograph	1015
85	With whom living	1078
103	Starting salary - monthly	3952
104	Pay on October 1 - monthly	4006
107	Race	336

Various approaches to the missing data problem have been proposed.[44] Where the number of respondents with missing data is small, all respondents with missing data may be eliminated from the analysis. This method is clearly inappropriate here, as it would drastically reduce the number of observations, to some extent unnecessarily, as the number of variables retained in the final analysis can be expected to fall considerably short of the number with which the analysis is begun. Alternatively, one may estimate the regression coefficients of $y = xb$ from the relationship:

(11) $$\mathrm{cov}(x_i, x_j)\hat{b} = \mathrm{cov}(x_i, y)$$

where $\mathrm{cov}(x_i, x_j)$ is the covariance matrix in which the $(ij)^{th}$ element is calculated on the basis of observations for which data on both i and j are available, and similarly for $\mathrm{cov}(x_i, y)$, and $\hat{b}$ is the vector of estimators. This method is more flexible in that it allows experimentation with all variables for all observations possible. It is particularly appropriate in attempting to arrive at a correct specification of an equation when there are a very large number of candidate variables. However, Haitovsky's Monte Carlo studies have shown that this method yields seriously biased results when the number of non-responses is high and the pattern particularly non-random. Thus, although this method was adopted in this study, the results must be regarded as provisional.

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[44] See Haitovsky.

As these estimates yield considerable insight on the correct specification of the educational production function, further studies, operating with far fewer variables, probably should adopt one of the many methods of assigning values to the missing data.

In Table 9, I present data on the number of respondents for which complete information was collected on each pair of the main variables used in the analysis of the Project Talent data for Blacks in U.S. Office of Education regions 1, 2, and 3. No analogous data is available for the data from the EEOS.

TABLE 9

Response Frequency for Each Pair of Variables
1960 Black Male Seniors in USOE Regions 1, 2, 3
in Project Talent Five-Year Follow-Up

Total Respondents 207

variable number / variable name and number	39	52	44	53	31	32	33	34	45
Dependent Variables									
39 Reading Comprehension	201	196	185	148	196	196	196	196	181
52 Math II & III		198	181	148	193	193	193	193	179
44 General Academic Ability			185	136	182	182	182	182	169
53 Starting Monthly Salary				154	149	149	149	149	139
Home Variables									
31 Father's Occupation					201	201	201	201	186
32 Mother's Occupation						201	201	201	186
33 Father's Education							201	201	186
34 Mother's Education								201	186
45 Own Room, Desk, Typewriter									186

table continued on following pages

	46	47	51	56	2	4	7	11	12	14	20	24	26
39	184	186	173	201	201	201	199	201	196	177	201	198	183
52	181	183	170	198	198	198	153	198	193	174	198	195	180
44	171	173	162	185	185	185	183	186	180	163	185	183	167
53	142	144	133	154	154	154	196	154	150	136	154	152	140
31	189	190	178	201	201	201	199	201	197	177	201	198	183
32	189	190	178	201	201	201	199	201	197	177	201	198	183
33	189	190	178	201	201	201	199	201	197	177	201	198	183
34	189	190	178	201	201	201	199	201	197	177	201	198	183
45	186	186	170	186	186	186	184	186	183	163	186	183	170

TABLE 9 (Continued)

Table 9 (continued)

	46	47	51	56	2	4	7	11	12	14	20	24	2[illegible]
46 Appliances	189	189	172	189	189	189	187	189	186	166	189	186	17[illegible]
47 TV, Telephone, Radio, Phonograph		191	173	191	191	191	189	191	188	168	191	188	17[illegible]
51 With Whom Living			178	178	178	178	176	178	177	157	178	175	16[illegible]
56 Race				207	207	207	205	207	202	183	207	204	18[illegible]
School ol Variables													
2 Class Size, Sci & Math					207	207	205	207	202	183	207	204	189
4 Senior Class Size						207	205	207	202	183	207	204	189
7 Educational Innovation							205	205	200	181	205	202	187
11 Starting Salary, Male, B.A.								207	202	183	207	204	189
12 Percent Teachers Fully Certified									202	178	202	199	184
14 Percent College Prep										183	183	180	176
20 Teacher's Graduate Training/Class											207	204	189
24 Tracking												204	186
26 Percentage Negro													189

VIII. PROVISIONAL RESULTS: THE IMPORTANCE OF SOCIAL CLASS AND THE SCHOOL

I have chosen three different measures of output: reading comprehension, intermediate high school mathematics competence, and a composite score based on a number of tests. The reading comprehension test measures what is commonly known as 'academic intelligence,' and is a good predictor of school success in an academic or liberal arts curriculum. The mathematics score is the sum of two test results, Math II and Math III. The first measures achievement in the mathematics generally offered up to and including the ninth grade. Math III covers topics normally included in tenth to twelfth grade mathematics courses, particularly in college preparatory curricula. Whereas the material in the reading comprehension test could easily be acquired outside the school, it seems reasonable to assume that the abilities measured in the two mathematics tests are learned in the classroom. The composite test score -- General Academic Aptitude -- is based on nine individual tests, as listed in Table 10.

All of the variables appearing in the following equations are listed in Table 11 along with their means and standard deviations. A table of zero order correlations appears in the appendix.

Table 10

Tests Used as Basis for Composite Score -- General Academic Ability

	weight in composite score
Mathematics	.08
Vocabulary I & II	.04
English Total	.28
Reading Comprehension	.26
Creativity	.06
Abstract Reasoning	.04
Math I	.12
Math II	.18
Total	1.00

Means and Standard Deviations of Variables
Black Male Twelfth Grade, USOE Regions 1,2,3

Variable Name	Mean	Standard Deviation
Dependent Variables		
Reading Comprehension	28.07	11.54
General Academic Ability	473.39	120.14
Math II and III	.84	.72
Starting Monthly Salary	369.12	189.28
Home Variables indicies		
Father's Occupation	6.07	4.20
Mother's Occupation	4.77	4.52
Father's Education	3.52	2.58
Mother's Education	3.64	2.20
Own Room, Desk, Typewriter	11.51	1.00
Appliances	114.76	12.45
TV, Telephone, Radio, Phonograph	13.22	1.11
With Whom Living	.27	.44
School Variables		
Class Size, Science and Math	28.77	3.94
Senior Class Size	447.48	286.02
Educational Innovation Index	8.94	1.04
Starting Salary, Male, B.A.	4448.07	438.92
% Teachers Fully Certified	96.47	11.03
% in College Prep (plus 100)	136.17	23.00
Teachers Grad Training/Class	.49	.59
Tracking	.86	.34
% Negro	45.87	38.54

The estimate of the educational production function for each of these output measures appears in Tables 12-14. The following aspects of the results are important.

The importance of social class: In each equation measures of the student's family background are highly significant. Two variables appear in all three equations -- the occupation of the father, and a measure of consumer durables in the home. The occupation of the father is the value of an occupation index scaled according to the mean income in particular occupations. Table 15 describes the scaling method. The consumer durables variable is the sum of yes responses to questions concerning the presence of a television, radio, telephone, and phonograph in the home. Both the father's occupation and the consumer durable variables are measures of family income. It is interesting to note that these income proxies explain scholastic achievement better than variables relating to the parents' education. Only in one of the three equations is a measure of the parents' education -- the mother's -- significantly related to achievement.

As expected, the family background variables are more closely associated with achievement in reading comprehension than in intermediate mathematics. In Table 16 the sum of the beta coefficients relating to background as opposed to school characteristics are reported.

Table 12

An Educational Production Function
Dependent Variable is Reading Comprehension
Black Twelfth Grade Students

Independent Variable	Regression Coefficient (t in parentheses)	Beta
1. Father's Occupation	0.5926 (3.4009)	0.2157
2. Mother's Education	0.5796 (1.7158)	0.1103
3. TV, Telephone, Radio, Phonograph	3.0166 (4.5938)	0.2899
4. Teacher's Graduate Training/Class	2.0403 (1.6698)	0.1038
5. Class Size, Science and Math	-0.4050 (-2.0607)	-0.1384
6. Tracking	-3.6627 (-1.5877)	-0.1092

Constant	-3.7117 (-0.3792)
R^2c	0.2444
$\lvert X'X \rvert$	0.6493

number of observations see Table 9

Table 13

An Educational Production Function
Dependent Variable is Mathematical Achievement

Black Twelfth Grade Students

Independent Variable	Regression Coefficient (t in parentheses)	Beta
1. Father's Occupation	0.0267 (2.4970)	0.1562
7. Educational Innovation	0.1164 (2.6506)	0.1684
5. Tracking	-0.2533 (-1.9089)	-0.1213
2. TV, Telephone, Radio, Phonograph	.1986 (4.8456)	0.3065
6. Expenditure per Student on Non-Teaching Inputs[a]	.0007 (2.6617)	0.1711
4. Teachers with Graduate Training/Class	.2227 (2.8029)	0.1820
3. Age of Building	-0.0069 (-2.4809)	-0.1559

Constant	-2.8981 (-4.1429)
R^2c	0.2199
$\lvert X'X \rvert$	.7747

number of observations see Table 9

[a]Expenditure per student on non-teaching inputs is a measure of expenditure per student minus a measure of the per student starting salary of a male fully certified teacher.

Table 14

An Educational Production Function
Dependent Variable is General Academic Ability

Independent Variable	Regression Coefficient (t in parentheses)	Beta
1. Father's Occupation	5.9546 (3.4707)	0.2082
7. Educational Innovation	8.5838 (1.2645)	0.0742
5. Tracking	-47.1432 (-2.0648)	-0.1350
2. TV, Telephone, Radio, Phonograph	41.9373 (6.5982)	0.3870
3. Mother's Occupation	2.5075 (1.5752)	0.0943
6. Class Size, Science and Math	-3.9846 (-2.0278)	-0.1307
4. Teachers with Graduate Training/Class	26.7528 (2.1930)	0.1307

Constant	-63.7764 (-0.5501)
R^2c	0.3275
$\lvert X'X \rvert$	0.6274

number of observations see Table 9

Occupational Scale

	TALENT RESPONSE LETTER AND DESCRIPTION	CENSUS TITLE	MEDIAN EARNINGS a/ AGE 45 - 55.	RECODE
A	Farm or Ranch Owner and/or Manager	Farmers and Farm Managers	$2381	24
B	Farm or Ranch Foremen	All Other Farm Workers and Foremen	$3486	35
C	Farm or Ranch Worker	Farm Laborers and Fore-men	$1496	15
D	Workman or Laborer	Laborers Except Farm and Mine	$2392	24
E	Private Household Worker	All Other Service Workers	$3328	33
F	Protective Worker	Protective Service Workers	$5069	51
G	Service Worker	Service Workers, Inc. Private Household	$3749	37
H	Semi-skilled Worker	Operatives and Kindred Workers	$4697	47
I	Skilled Worker or Foreman	Craftsmen, Foremen and Kindred Workers	$5450	55
J	Clerical Worker	Clerical and Kindred Workers	$5369	54
K	Salesman	Sales Workers	$5855	59
L	Manager	Managers, Officials, and Proprietors Except Farm	$7232	72
M	Official	Managers, Officials and Proprietors Except Farm	$7232	72

TALENT RESPONSE LETTER AND DESCRIPTION	CENSUS TITLE	MEDIAN EARNINGS AGE 45 - 55	RECODE
N Proprietor or Owner	Managers, Officials and Proprietors Except Farm	\$7232	\$72
O Professional	Professional, Technical, and Kindred Workers	\$8056	\$81
P Technical	Profession, Technical, and Kindred Workers	\$8056	\$81

a/ Source: United States Census Population: 1960. Subject Reports. Occupation by Earnings and Education. United States Department of Commerce, Bureau of the Census. Washington, D.C., 1963.

Earnings refer to males.

Table 16

Comparison of Social Class and School Variables in the Production Functions

	dependent variable		
	Reading Comprehension	Mathematics	Composit Score
sum of beta coefficients for social class variables[a]	0.6159	.4627	.6895
sum of beta coefficients for school variables[a]	0.3514	.7937	.4706

[a]The variables used are those which appear in Tables 12-14.

The importance of the school.

a. Teacher Quality. A variable measuring teacher quality -- the number of teachers with graduate training divided by the number of classes in the school -- is significantly related to scholastic achievement, although in the production of reading comprehension the relationship is rather weak. This variable is such a poor measure of what makes a good teacher that our results give us only the vaguest idea of the importance of teacher quality as a determinant of scholastic achievement. Our regression coefficients are surely underestimates.

b. Class Size. The average size of classes in the science and mathematics subjects is related to scholastic achievement for two of our three cases. The variable is, of course, a general measure of class size, being correlated ($r = .4522$) with class size in other subjects. Nonetheless, it seems somewhat anomolous that the class size variable is not significantly related to mathematics achievement.

c. Educational Innovation. An index of educational innovation was constructed to measure the extent of innovation in both curriculum and equipment. The index is based on the responses to three items on the General School Characteristics Questionnaire which was completed by the school principal or his staff:

1. "In which of the following areas has your grades 9-12 school taken part in a large scale inter-system tryout of a special experimental curriculum?" (response by subject area, or "none")

2. "In which of the following areas has your grades 9-12 school or school system developed and tried out its own special experimental curriculum?" (response by subject area, or "none")

3. "Which of the following statements best describes the current use (grades 9-12) of teaching machines in your school? (Teaching machines may be thought of as individual self-instructional devices which automatically provide both learning material and answers to student responses. They do not include the usual educational films, slides, educational TV, etc.)" (response by level of use, including "none")

The "none" responses to the above three questions were summed and subtracted from ten to yield our index of educational innovation. In one of our three cases, educational innovation is significantly related to achievement. It will be seen below that in a second case (the production of general academic ability), the educational innovation variable is significant in a more fully specified equation. It is likely that some of the apparent influence of educational innovation is a reflection of general school atmosphere, the innovative schools being more open and experimental generally, not simply with respect to curriculum and equipment.

4. Economies of Scale. In no case did the addition of a variable measuring the senior class size yield a statistically significant increase in the fraction of variance explained by the equation. Table 17 records the t-statistic for the class size variable when added to each equation, along with the size of the estimated coefficient. Similar results were generated using the EEOS data. The fact that the estimated effect of changes in senior class size is insignificantly different from zero is in conflict with much of the literature on economies of scale in secondary schooling.[45] It should be noted that the senior class size is a reasonably good proxy for size of community. Thus our result may reflect a combination of genuine economies of scale which are offset by unmeasured negative effects of the center city school environment. A non-linear relationship between senior class size and achievement would seem plausible even if the community size could be accurately controlled in the equation. I have not estimated the relationship with a non-linear senior class size variable.

5. School policy: tracking. The measure of school tracking is a dummy variable indicating that tracking exists if the school has two or more tracks. (A single track with electives is not regarded as tracking.) In all three cases, tracking is negatively associated with scholastic achievement. The predicted

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[45] See the work of J. Riew, E. Cohn, and H. J. Kiesling.

Table 17

t-Statistics for Senior Class Size Variable

Dependent Variable	t-statistic	Estimated Effect of Increasing Class Size on Achievement
Mathematical Achievement	0.6927	+
Reading Comprehension	0.1464	-
General Academic Ability	1.4199	+

level of scholastic achievement in schools with tracking is .3174 .3518, and .4085 standard deviations below those which do not on the reading, mathematics, and composite score, respectively. Two interpretations of this result come to mind. Both begin from the presumption that black students are likely to be on the average lower achievers, and of lower social class than their white school mates. The first interpretation is that tracking has a negative effect on scholastic achievement in that it minimizes the contact between the students in our sample and the higher achieving higher class students in the rest of the school. This interpretation can be tentatively rejected, as we would in this case expect that the achievement and/or class composition of the school would produce a positive effect on achievement, holding constant the degree of tracking. That this is not the case is indicated by the fact that, when added to the above equations, the variable measuring the percentage of children in the school in the college preparatory subjects is never significantly related to achievement.

A more compelling explanation is simply that the level of resources devoted to a child's education is not uniform within a school -- it varies in part according to the track the student is in. This being the case, and recalling that blacks are disproportionately likely to be placed in the "slow" or otherwise disadvantaged tracks, we may interpret the negative coefficient of tracking as reflecting the influence of unequal school resources within schools. In schools with tracking, the black students, on the average, receive a level of school resources which falls short of the school average. This is reflected in their achievement scores.

6. School Policies: Integration. When we add a variable measuring the percentage of the student body which is black, we get the results in Tables 18 through 20. In two of the three cases there is a significant negative relationship between the level of achievement by our sample of black students and the proportion of the student body which is black. Given the fact that a measure of the social class and achievement levels of the school (percentage in college preparatory subjects) is not significantly related to black achievement, it is difficult to interpret this result as a peer effect involving the transfer of "good" learning habits, language models, etc., from the high achieving whites to the low achieving blacks. An alternative (untestable) interpretation is that the apparent impact of the proportion of blacks in the school arises from the fact that the social backgrounds of black children in integrated schools and those in all black, or nearly all black, schools differ in ways which are relevant to learning but which are not captured in our crude social class measures. The results cannot be interpreted as suggesting that school integration will raise black achievement.

Table 18
An Educational Production Function
Dependent Variable, Reading Comprehension
with Integration Variable
Black Twelfth Grade Students

Independent Variable	Regression Coefficient (t in parentheses)	Beta
1. Father's Occupation	0.5503 (3.1162)	0.2003
2. TV, Telephone, Radio, Phonograph	2.9622 (4.5124)	0.2846
3. Mother's Education	0.6634 (1.9363)	0.1262
4. Teacher's Graduate Training/Class	1.9449 (1.5924)	0.0989
5. Tracking	-3.7356 (-1.6221)	-0.1114
6. Class Size, Science & Math	-0.4111 (-2.0959)	-0.1405
7. Percentage Black	-0.0253 (-1.3636)	-0.0846

Constant	-1.5930 (-0.1611)
R^2c	.2476
$\|X'X\|$	.6127
number of observations	see Table 9

Table 19
An Educational Production Function
Dependent Variable is Mathematic Achievement
with Integration Variable
Black Twelfth Grade Students

Independent Variable	Regression Coefficient (t in parentheses)	Beta
1. Father's Occupation	0.0237 (2.2301)	0.1384
2. TV, Telephone, Radio, Phonograph	0.1912 (4.7183)	0.2951
3. Age of Building	-0.0044 (-1.5228)	-0.1000
4. Teacher's Graduate Training/ Class	0.2115 (2.6952)	0.1728
5. Tracking	-0.2643 (-2.019)	-0.1266
6. Class Size, Science & Math	0.0006 (2.4513)	0.1561
7. Educational Innovation	0.1504 (3.3212)	-0.0404
8. Percentage Black	-0.0033 (-2.5726)	-0.1751

Constant	-2.9744 (-4.3076)
R^2_c	0.2413
$\lvert X'X \rvert$	0.6126
number of observations	see Table 9

Table 20
An Educational Production Function
Dependent Variable is General Academic Ability
with Integration Variable
Black Twelfth Grade Students

Independent Variable	Regression Coefficient (t in parentheses)	Beta
1. Father's Occupation	5.3193 (3.1216)	0.1800
2. TV, Telephone, Radio, Phonograph	41.0042 (6.5539)	0.3788
3. Mother's Education	2.9140 (1.8522)	0.1096
4. Teachers with Graduate Training/Class	25.9364 (2.1600)	0.1267
5. Tracking	-51.0832 (-2.2691)	-0.1463
6. Class Size, Science & Math	-3.8132 (-1.9711)	-0.1251
7. Educational Innovation	14.2960 (2.0433)	0.1236
8. Percentage Black	-0.5107 (-2.7441)	-0.1638

Constant	-78.8240 (-0.6901)
R^2_c	0.3488
$\|X'X\|$	.5536
number of observations	see Table 9

IX. COMPARISON OF PROJECT TALENT AND EQUALITY OF EDUCATIONAL OPPORTUNITY SURVEY RESULTS

In order to assess the generality of these results, I will now estimate similar functions with a different body of data. The following estimates are for black students who were enrolled in the twelfth grade in the fall of 1965. The data were collected by the U.S. Office of Education as part of the Equal Educational Opportunity Survey. Some of the results of this survey have been reported in Equality of Educational Opportunity, known popularly as the Coleman Report, after its principal author.[46] The sample and a number of serious shortcomings of the data are described in detail elsewhere. Any reader adventuresome enough to take seriously my preliminary results is urged to consult these sources.[47]

The variables available for the empirical estimation of the educational production function along with their means and standard deviations appear in Table 21. A table of the zero order correlation coefficients appears in an appendix. Unfortunately, a number of variables available in the Talent sample are not measured in the EEOS data and vice versa. The estimate of our basic equation appears in Table 22. A number of comments are in order.

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[46] Coleman, *et al.* Our estimations are based on the correlation tables and means and standard deviations of each variable, as reported in Vol. II of Equality of Educational Opportunity.

[47] In addition to the Report itself, see Bowles and Levin (1968a), Hanushek, and Hanushek and Kain.

Table 21

Means, Standard Deviation and Zero Order Correlations Among Variables Used in Estimates

Variable[a]	Mean	Standard Deviation
Dependent Variable:		
Verbal Achievement Scale Score	49.2202	14.4512
Home Environment:[b]		
Reading Material in the Home	-0.1091	0.6159
Number of Siblings (positive = few)	-0.3334	1.0275
Parents' Educational Level	-0.1672	0.8389
School Environment:		
Teacher's Verbal Ability Score	21.2211	2.5593
Science Lab Facilities (index)[c]	89.4083	22.4557
Average Time Spent in Guidance	1.8528	0.7847
Number of Days in Session	179.8984	4.1359
Size of the Senior Class	264.3718	212.7663
Student Attitudes:		
Sense of Control of Environment[e]	-0.1265	0.7654
Self Concept[f]	0.0460	0.7132

Notes to Table 21

a) Further definition of these variables, as well as the survey instruments on which they were based, is available in Coleman (14).

b) The home environment and student attitude variables have been normalized to mean = 0 and standard deviation = 1 for the national sample taken as a whole.

c) Range = 0-99. A score of 33, 66, or 99 indicates that the school has one, two, or all of the following types of labs: biology, chemistry, and physics.

d) The verbal ability score is based on the School and College Ability Test Scores of the Educational Testing Service.

e) The sense of control variable is based on the student agreement or disagreement with three statements: Good luck is more important than hard work for success; Every time I try to get ahead, something or somebody stops me; and People like me don't have much of a chance to be successful in life.

f) The self concept variable is based on the student's responses to the following items: How bright do you think you are in comparison with the other students in your grade? Sometimes I feel that I just can't learn (agree - disagree); I would do better in school work if teachers didn't go so fast (agree - disagree).

appears in Table 22. A number of comments are in order.

First, the parameters are roughly consistent with our earlier results. The very significant estimate of the influence of teacher quality (as represented by teachers' verbal abilities score) is particularly important, and not surprising when we recall that the teacher is by far the single most important school input.[48]

The importance of teacher quality has been confirmed by much of the current work in the estimation of educational production functions.[49]

Given our findings from the Talent sample, the absence of a class size variable is surprising. The failure of a class size variable to appear in the equation may be a reflection of severe errors in the measurement of this variable.[50] Although class size does not appear to be a significant influence on achievement in a number of studies (for example, Hanushek, and Levin), at least one author, Kiesling, found a highly significant relationship between students-per-teacher and achievement.[51]

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[48]The teacher's verbal ability test consists of only 30 questions and is self-administered. If, as seems likely, the variance of the error component in this measure is large, the estimate of the associated regression coefficient may be seriously downwardly biased. The same reasoning, of course, applies to the other school inputs.

[49]See Kiesling, Hanushek. In addition to these results, Levin found that two measures of teacher qualtiy (verbal score and type of college attended) were highly significant in explaining verbal achievement among sixth grade black students of the third socio-economic quartile in a large metropolitain area.

[50]See Bowles and Levin (1968b).

[51]The negative relationship between teacher-student ratio and a

TABLE 22

An Educational Production Function

Black Twelfth Grade Students

Independent Variable (the dependent variable is verbal achievement)	Regression Coefficient (t in parentheses)	Beta
1. Reading Material in the home	2.0931 (2.8270)	0.0892
2. Number of Siblings (positive-few)	1.1812 (4.2513)	0.1288
3. Parents' Educational Level	2.4213 (4.3870)	0.1406
4. Teacher's Verbal Ability Score	1.2462 (7.1445)	0.2207
5. Science Lab Facilities	0.0505 (2.5837)	0.0785

Constant	19.4946 (5.1938)
R_c^2	0.1684
$\|X'X\|$	0.6761
number of observations	1,000

The absence of a measure of school policy, reflecting in part the quality of the interaction between students and teachers, is to be explained by the profusion of imperfect measures of this dimension of the input structure. When we entered eleven of the school policy variables into the above equation, we could not accept the hypothesis that all of the regression coefficients for these variables were zero.[52] In order to represent the influence of this set of variables, we have introduced a variable representing the extent of guidance counselling in the school as a rough proxy. We have further added a variable chosen to represent the general level of community interest in and support of education, days in session. The resulting equation appears in Table 23.

Both variables are highly correlated with measures of overall

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51 cont measure of school output found by Welch may be a reflection of the smaller classes in rural schools and the failure to take account of the negative influences on learning associated with a rural home and community environment. (The positive association between teacher-student ratio and tenth grade verbal scores in twenty-two Atlanta public schools estimated by J. W. Holland and J. Burkhead is difficult to interpret, as the equation in which this finding is reported includes a measure of per pupil expenditure (plus a number of insignificant variables).) This seems to suggest that even with a given level of expenditure, reduction in class size produces sufficiently strong effects on achievement to more than offset the associated opportunity costs.

52 The F value leading to the rejection of the hypothesis was 2.39 with 11 and 984 degrees of freedom. Thus the hypothesis was rejected at the 99% level of significance.

TABLE 23

Educational Production Function

with School Policy and Community Support Proxies

Black Male Twelfth Grade Students

Independent Variable (dependent variable is verbal achievement)	Regression Coefficient (t in parentheses)	Beta
1. Reading Material in the Home	1.8254 (2.4632)	0.0778
2. Number of Siblings (positive = few)	1.7184 (4.0405)	0.1222
3. Parents' Educational Level	2.4083 (4.3787)	0.1398
4. Teacher's Verbal Ability Score	1.0348 (5.5186)	0.1833
5. Science Lab Facilities	0.0375 (1.8871)	0.0582
6. Average Time Spent in Guidance	1.4636 (2.3364)	0.0795
7. Days in Session	0.2054 (1.9396)	0.0588

Constant	-14.5708 (-0.7706)
R^2_c	0.1780
$\|X'X\|$	0.4569
number of observations	1,000

support for education, such as the level of teachers' salaries and system-wide expenditure per pupil. Both are also positively associated with school policy variables such as the extent of extra curricular activities and foreign language courses, though 'days in session' is much less closely associated with these school variables than is the guidance counselling measure.[53]

How similar are these findings to those based on the Project Talent sample? In order to answer the question, consider the two equations predicting comparable outputs: the Talent equation for reading comprehension (Table 12), and the EEOS equation in Table 23. Although comparison is difficult because the set of variables in each equation is not the same, note that the estimated impact of increasing all school inputs by a standard deviation is virtually identical in both equations. The impact for the Talent and EEOS equations is .35 and .32 standard deviation units in the dependent variable, respectively.[54] The estimated impact of changes in

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[53] Of course, the days in session measure may simply reflect urban-rural differences, as we have considerable evidence that rural schools are open for fewer days per year. (See Coleman) As a test of this hypothesis, we added a variable measuring the size of the senior class to the equation. This new variable was insignificant, and, although its introduction lowered the estimated regression coefficient for 'days in session' by about ten percent, the latter variable was still significantly different from zero at the 95% significance level. The remainder of the equation was altered only slightly.

[54] This figure is the sum of the beta coefficients for the school inputs; the days in session variable is regarded as a community variable, not a school input.

social background is roughly similar in both equations (.62 standard deviation units for the Talent data and .40 for the EEOS), as is the portion of variance explained in both equations (0.2444 for Talent and 0.1780 for the EEOS).

Some further support for the stability of these findings is contained in Tables 24 and 25, which present equations estimated from the EEOS data for samples of Southern and Northern students separately. (These samples are different from the national EEOS sample used thus far.) Taking into account the numerous shortcomings of the data and methods used, the similarity in results is striking. Although the above evidence is much too weak to demonstrate the generality of the Project Talent results, none of the available information is seriously inconsistent with the general qualitative outlines of the functions based on Talent data.

Table 24

Estimated Regressions for Samples of Northern 12th Grade Students

Independent Variable (dependent variable is verbal achievement)	Regression Coefficient (t in parentheses)	Beta
1. Reading Material in the Home	1.279 (1.6013)	.052
2. Number of Siblings (positive = few)	1.660 (3.700)	.116
3. Parents' Educational Level	2.655 (4.626)	.151
4. Family Stability	.899 (1.675)	.051
5. Teacher's Verbal Ability Score	.721 (3.193)	.097
6. Science Lab Facilities	.059 (2.137)	.067
7. Days in Session	.189 (1.971)	.062

Constant	-2.585 (-0.1462)
R^2_c	.090
$\lvert X'X \rvert$	.730
number of observations	1,000

Table 25

Estimated Regressions for Samples of Southern 12th Grade Students

Independent Variable (dependent variable is verbal achievement)	Regression Coefficient (t in parentheses)	Beta
1. Reading Material in the Home	1.841 (2.629)	.083
2. Number of Siblings (positive = few)	1.794 (4.438	.135
3. Parents' Educational Level	2.185 (4.181)	.132
4. Family Stability	.823 (1.858)	.053
5. Teacher's Verbal Ability Score	1.097 (6.593)	.210
6. Science Lab Facilities	.027 (1.724)	.052
7. Average Time Spent in Guidance	2.017 (3.266)	.102

Constant	20.373 (6.247)
R^2_c	.1961
$\|X'X\|$	.519
number of observations	1,000

X. SPECIFICATION BIAS[55]

Thus far we have been working with a model in which no explicit account is taken of student endowments at the beginning of school. The biases in our estimates resulting from this exclusion are suggested by the following exercise. We have attempted to explain a similar achievement score at grade one by our set of explanatory variables. The resulting equation and the calculation of the specification bias appear in Table 26. At the first grade level, the school input variables were never significantly different from zero (at conventional levels).

Given the crudeness of both the measures and the technique, the particular numerical estimates are subject to considerable error. Nonetheless, as expected, the apparent influence of social class on school learning is drastically reduced, while the significance of school inputs is not affected. Of course, as long as we use an additive linear model with no interaction effects and plausibly assume no effect of school inputs on initial scores, there can be no estimated bias of the school inputs. Then our result is hardly surprising.

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[55]The analysis in this section is restricted to the EEOS data, as the Project Talent data at my disposal do not include grade one scores or the relevant student attitude measures.

Table 26

Correction For Specification Bias Due to Omitted Initial Endowments in the Educational Production Function.

Black Twelfth Graders.

	Regression Coefficient at grade twelve (1)	at grade one[a] (2)	Corrected Regression Coefficients[b] (3)
Home Environment:			
Reading Material in the Home	1.8254	.348 (1.97)	1.1969
Number of Siblings (positive = few)	1.7184	--	1.7184
Parents' Educational Level	2.4083	.884 (5.85)	.812
School Environment:			
Teacher's Verbal Ability Score	1.0348	--	1.041
Science Lab Facilities	.0375	--	.0375
Average Time Spent in Guidance	1.4636	--	1.4636
Days in Session	.02054	--	.02054

a) t ratios are in parentheses. The coefficient of determination for the equation was .05.

b) Column (3) = Column (1) - Column (2) x $b_{1.12}$, where $b_{1.12}$, the regression coefficient of A_1 in equation (7), is assumed to be $.5\left(\frac{\sigma_{12}}{\sigma_1}\right)$ or 1.806.

Recall that equation (2) represented our reduced form. Yet a complete specification of the learning envisonment must plausibly include student motivations. In our case these attitudes are represented by two measures, student self-concept, and student sense of control over environment.[56] When these are added to the basic equation, we arrive at the equation presented in Table 27.

It is worth noting that the structural parameters related to the school inputs change very little, suggesting that, in this case, the simultaneous equation bias is relatively small. The attitude variables are powerfully related to achievement -- the proportion of variance explained is almost doubled by their inclusion.

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[56]See Table 21 for the measurement of these variables.

TABLE 27

Educational Production Function with Student Attitudes Measured

Black Male Twelfth Grade Students

Independent Variable (dependent variables is verbal achievement)	Regression Coefficient (t in parentheses)	Beta
1. Reading Material in the Home	0.5686 (0.8243)	0.0242
2. Number of Siblings (positive = few)	1.5091 (3.8459)	0.1073
3. Parents' Educational Level	1.8527 (3.6390)	0.1075
4. Science Lab Facilities	0.0355 (1.9392)	0.0552
5. Days in Session	0.1821 (1.8653)	0.0521
6. Teacher's Verbal Ability Score	1.1069 (6.3977)	0.1960
7. Average Time Spent in Guidance	1.7673 (3.0523)	0.0960
8. Student's Control of Environment	4.4418 (8.3100)	0.2353
9. Student's Self-Concept	4.2767 (7.4531)	0.2111

Constant	-12.2473 (-0.7020)
R^2c	0.3031
$\|X'X\|$	0.3870
number of observations	1,000

XI. CONCLUSION: THE EFFECTS OF SCHOOLING ON ACHIEVEMENT

The imperfect measurement, the limited exposure to the educational environment, and our fundamental ignorance about how children learn establish the presumption that the estimated effect of different schools will be quite limited. Nonetheless, our estimated equations suggest that the difference in achievement between students in schools with inputs at levels one standard deviation below the mean for our sample compared with students in schools one standard deviation above the mean ranges from .64 to 1.5974 standard deviations on our achievement scale.[57]

Given the limited nature of the sample, and the inadequate opportunity to explore the available data, I will refrain from generalizing from these initial, encouraging results. What we have uncovered so far, however, suggests that our emphasis on the school in the study of human capital formation and income distribution is not misplaced. It is certainly the most important learning environment directly under social control. We have identified a number of school inputs which do seem to affect learning.

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[57] For the purposes of these calculations, length of school year is considered a community variable, not a school inputs.

XII. POSTSCRIPT: INITIAL RESULTS CONCERNING THE EFFECT OF SCHOOL QUALITY UPON EARNINGS

In the analysis of the effect of schooling on economic growth and the distribution of income, the relationship between education and earnings plays a central role. Studies of this relationship have ordinarily measured education by years of schooling.[58] Recently a number of authors have investigated the relationship between scholastic achievement and earnings.[59]

Here I adopt a different approach, namely the investigation of the relationship between the level and quality of school inputs, on the one hand, and earnings in post-school employment, on the other.[60] To isolate the importance of the particular dimensions of school inputs, I deal with students all of whom reached the twelfth grade.

Our data on earnings are for the first job after leaving school, as reported by the student, and are undoubtedly considerably in error. Evidence of bias is clear from the fact that the reported monthly earnings of respondents in this sample are 1.67 times the earnings of the analogous group as reported in the U.S. census. Thus the findings below should be regarded as exploratory hypotheses for further

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[58] Hanoch.

[59] Weiss, and Weisbrod, Hansen and Scanlon.

[60] A similar approach has been used by Welch. The results reported are based entirely on the Project Talent data.

study.[61] Preliminary analysis of the data on the Project sample of blacks in the U.S. Office of Education regions 1, 2, and 3, and who were seniors in 1960 resulted in the equation which appears in Table 28. Note the following:

First, the relationship between social background and earnings in the first job is very weak. (See Table 29.)

Second, there is no statistically significant relation between the measures of scholastic achievement and earnings. The t-statistics for each test score when added to the above equation appears in Table 30.[62]

Third, some school inputs are significantly related to earnings, even for individuals with the same number of years of schooling.

The above findings suggest an interesting and important proposition for further study. They suggest first that schools perform more than a selection function and produce a unique effect upon earnings, but, second, that this effect is not primarily

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[61] Annual earnings of the Project Talent respondents were calculated using ten months of employment per year. The discrepancy probably reflects both erroneous responses and the unrepresentative nature of the Talent sample.

[62] Hansen, Weisbrod, and Scanlon found a statistically significant relationship between the AFQT and earnings in a group of low-achieving Northern blacks. Although their findings heighten my skepticism concerning the above estimation, it should be kept in mind that the two population groups are not at all comparable. The mean years of schooling in the Hansen, Weisbrod, and Scanlon sample is nine years and measured achievement is around the third grade equivalent level.

Table 28

An Educational Production Function: Dependent Variable is Starting Monthly Salary Blacks Who Were Twelfth Grade Students in 1960

Independent Variable	Regression Coefficient (t in parentheses)	Beta
1. Educational Innovation	33.62 (2.5988)	.1845
2. Percent in College Prep	1.50 (2.5921)	.1827
3. Part-Time Teachers/Total Classes in School	-345.04 (-1.9349)	-.1461
4. Class Size, Science & Math	-6.09 (-1.6715)	-.1269

Constant	57.9877 (0.2783)
R^2c	.0612
$\|x'x\|$	.7200
number of observations	see Table 9

Table 29

Relation of Socio-Economic Status of Parents to Earnings
t-Statistics for SES Variables

Socio-Economic Status Variable	t-Statistic[a]	Affect of Higher Status on Earnings
1. Father's Occupation	.0804	-
2. Mother's Occupation	1.1442	-
3. Father's Education	1.3793	+
4. Mother's Education	.5989	+
5. Own Room, Desk, Type-writer	.0927	+
6. Appliances	.3939	+
7. TV, Telephone, Radio, Phonograph	.4220	-
8. With Whom Living	1.3147	+

[a]The t-statistics reported here are for each SES variable when added to the equation in Table 28.

Table 30
Relation of Scholastic Achievement to Earnings
t-Statistics for Test Scores
Dependent Variable is Starting Monthly Salary
Blacks Who Were Twelfth Grade Students in 1960

Test	t-statistic[a]	Affect of Higher Achievement on Earnings
Reading Comprehension	.1151	+
Mathematics	.1092	+
General Academic Ability	.1325	-

[a]The t-statistics reported here are for each test score when added to the equation in Table 28.

conveyed through cognitive development (as measured in scholastic achievement scores), but through other effects of schooling on earnings capacities.[63]

If the proposition is correct, we must renew the search for adequate economically relevant measures of school output and substantially revise our view of the role of education in the production process. The proposition will be tested with other bodies of Project Talent data in my forthcoming work. The above results by themselves are no more than suggestive.

[63] A strong case for the second part of this proposition is made by Herbert Gintis in his, "Toward a Method in the Economics of Education -- The Educational Production Function," unpublished mimeo, Harvard University, 1969.

Table 31

Zero Order Correlations Among Variables
(EEOS data)

Variables	1	2	3	4	5	6	7	8	9	10	11
Verbal Achievement Scale Score	1.000	.220	.225	.261	.308	.188	.254	.153	.262	.350	.28
Reading Material in the Home		1.000	.178	.363	.204	.152	.218	.106	.216	.161	.11
Number of Siblings			1.000	.263	.152	.122	.134	.108	.173	.089	.04
Parents' Educational Level				1.000	.198	.136	.174	.050	.205	.112	.11
Teacher's Verbal Ability Score					1.000	.280	.453	.179	.506	.054	-.05
Science Lab Facilities						1.000	.295	.170	.263	.078	-.03
Average Time Spent in Guidance							1.000	.287	.522	.063	-.07
Number of Days in Session								1.000	.304	.058	-.02
Size of the Senior Class									1.000	.070	-.06
Sense of Control of Environment										1.000	.30
Self Concept											1.00

Source:

Coleman, Vol. II

Table 32
Zero Order Correlation Among Variables

variable number variable ream	1	2	3	4	5	6	7
1 Reading Comprehension	1.000	.5170	.8855	.0278	.2955	.2016	.2157
2 Mathematics		1.000	.7301	.0414	.2097	.1348	.1738
3 General Academic Ability			1.000	.0368	.2936	.2096	.2212
4 Salary				1.000	-.0105	-.0644	.1217
5 Father's Education					1.000	.2701	.4227
6 Mother's Education						1.000	.2992
7 Father's Occupation							1.000
8 Mother's Occupation							
9 Own Room Desk Typewriter							
10 Appliances							
11 TV, Telephone, Radio, Phonograph							
12 With Whom Living							
14 Class Size, Sci & Math							
15 Senior Class Size							
16 Educational Innovation							
17 Starting Salary, Male, B.A.							
18 Teachers Fully Certified							
19 Percent College Prep							
20 Teacher's Starting Salary							
21 Tracking							
22 Percentage Negro							

	8	9	10	11	12	14	15
1	.2515	.1212	.1334	.3704	-.0659	-.1970	-.1032
2	.1311	.0928	.1456	.3418	-.1229	-.1358	.0166
3	.2258	.1396	.1634	.4573	-.1331	-.2136	-.0262
4	.0525	-.0270	.0455	.0020	.0625	-.1154	.0524
5	.2830	.1230	.0852	.1621	-.2212	.0045	.0511
6	.3364	.1447	.0887	.0973	-.0059	-.0546	-.0521
7	.5524	.1932	.1647	.1677	-.1947	-.0458	.0607
8	1.000	.0207	.1888	.2286	-.0716	-.0322	.0325
9		1.000	.3215	.2934	-.1601	.0677	.0919
10			1.000	.4738	-.1325	-.0816	-.0976
11				1.000	-.1155	.0107	.0791
12					1.000	.0157	-.1382
14						1.00	.4522
15							1.00
16							
17							
18							
19							
20							
21							
22							

	16	17	18	19	20	21	22
1	-.0085	.0499	-.1330	.1323	.1826	-.2275	-.1088
2	.0979	.0093	-.0477	.1529	.2186	-.1749	-.1976
3	.0551	.0239	-.1007	.2166	.2190	-.2588	-.1660
4	.1761	.0348	.0574	.1338	.0003	-.0523	.1207
5	-.0686	.0508	-.1872	.1480	.0398	.0171	-.1412
6	-.0367	-.0412	-.0115	.1036	.1304	.0006	.0357
7	.0475	-.0208	-.1415	.1583	.0930	-.0638	.0980
8	.0252	.0650	-.0783	.0289	.0477	-.0413	.1220
9	-.0820	.1412	.0899	.0581	.1256	.0090	-.1295
10	.0138	.0298	.0090	.0504	.0394	-.0130	.0436
11	.0216	.1404	.0049	.1356	.0839	-.1200	-.0501
12	-.0134	-.0255	.0687	-.1143	-.0843	-.0673	.1367
14	-.1256	.1550	.2373	-.1405	-.1178	.4300	-.0366
15	.2926	.1370	.1190	.1829	-.2768	.3847	-.3215
16	1.000	.0253	.1120	-.1164	-.1384	.0599	.3012
17		1.000	.0647	-.3222	.0139	.1095	.0699
18			1.000	-.0902	-.1188	.0955	.2063
19				1.000	.0864	-.0484	-.3034
20					1.000	-.2230	-.0522
21						1.000	-.0268
22							1.000

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